

“PREDICTION OF ELASTIC PROPERTIES OF 2D ORTHOGONAL PLAIN WEAVE FABRIC COMPOSITE”

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF TECHNOLOGY
IN
MECHANICAL ENGINEERING
[Specialization: Machine Design and Analysis]

By

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NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA
ORISSA, INDIA
MAY, 2009

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CERTIFICATE

This is to certify that the work in this project report entitled “**Prediction of elastic properties of 2D orthogonal plain weave fabric composite**” by **Irshad Ahmad Khan** has been carried out under our supervision in partial fulfillment of the requirements for the degree of **Master of Technology** in *Mechanical Engineering* with “**Machine Design and analysis**” specialization during session 2008 - 2009 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

To the best of our knowledge, this work has not been submitted to any other University/Institute for the award of any degree or diploma.

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M-Tech (Machine Design and Analysis)

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ABSTRACT

A two dimensional analytical method has been developed for the prediction of elastic properties of 2D orthogonal plain weave fabric laminae. Strand undulation and continuity in the in warp and fill directions, actual strand cross-section and weave geometry, strand fiber volume fraction and possible gap between two adjacent strands have been considered in the analysis. The elastic properties of WF lamina are determined with assumption that classical laminate theory is applicable in the unit cell and the bending deformations of one unit cell are constrained by the adjacent unit cell in case plain weave fabric lamina. Here two material system T-300 carbon/epoxy and E-glass/epoxy were taken for investigation of elastic properties i.e. young modulus, Shear modulus, and Poisson ratio. MATLAB tool has been used for the purpose. Here Slice array model (SAM) has been taken for analysis and two paths (sinusoidal and circular) have been taken for determination of total compliance constant by averaging the local compliance constant. Effect of woven fabric geometrical parameters on the elastic properties of the laminae has been investigated. Good correlation is observed between the predicted elastic properties from MATLAB program and experimental results have been taken from previous literature.

Keyword: Plain weave fabric lamina; Prediction; Two dimensional; Elastic properties

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Nomenclature

a	Strand width
A_{ij}, i, j	In-plane stiffness constants of WF lamina
a_{yt}, z_{yt}	Gap parameters in X-Z plane
a_{xt}, z_{xt}	Gap parameters in Y-Z plane
$E_{fL}, E_{fT}, G_{fLT}, G_{fTT}, \nu_{fLT}$	Elastic properties of fiber along longitudinal and Transverse directions
$E_L, E_T, G_{LT}, G_{TT}, \nu_{LT}$	UD lamina elastic properties along the fiber and transverse fiber directions
E_x, E_y, G_{xy}, ν_x	Effective elastic properties of WF lamina
g	Gap between the adjacent strands
h	Maximum strand thickness
H_L	Total thickness of WF lamina
h_m	Thickness of matrix at $x=0, y=0$
h_t	Fabric thickness
$hy_1(y), hy_4$	Thickness of matrix in Y-Z plane at $x=0$
$S_{ij}(\theta), \overline{S_{ij}}, i, j = 1, 2, 6$	Local reduced and averaged compliance Constant
u	Undulated length in interlacing region
V	Volume
V_f	Fiber volume fraction
x, y, z	Cartesian coordinates
$zx_1(x, y), zx_2(x, y)$	Strand Shape parameters
$zy_1(y), zy_2(y)$	Strand Shape parameters
$\theta(x), \theta(y)$	Local off-axis angles of the undulated strand

θ Maximum off-axis angle of the undulated strand

Superscripts

o WF overall composite properties

pm Quantities of pure matrix

s Quantities of strand

Over bars indicates average values

Subscript

f Quantities in fill direction

L Quantities in fiber direction

T Quantities in transverse fiber direction

w Quantities in warp direction

Chapter 1

INTRODUCTION

INTRODUCTION

The increasing demand for lightweight yet strong and stiff structures has led to the development of advanced fiber-reinforced composites. These materials are used not only in the aerospace industry but also in a variety of commercial applications in the automobile, marine and biomedical areas. Traditionally, fibrous composites are manufactured by laminating several layers of unidirectional fiber tapes pre-impregnated with matrix material. The effective properties of the composite can be controlled by changing several parameters like the fiber orientation in a layer, stacking sequence, fiber and matrix material properties and fiber volume fraction. However, the manufacture of fibrous laminated composites from prepregs is labor intensive. Laminated composites also lack through-thickness reinforcement, and hence have poor inter laminar strength and fracture toughness.

A large variety of fibers are available as reinforcement for composites. The desirable characteristics of most fibers are high strength, high stiffness, and relatively low density. Glass fibers are the most commonly used ones in low to medium performance composites because of their high tensile strength and low cost. In woven fiber, fibers are woven in both principal directions at right angles to each other. Woven glass fibers is used to achieve higher reinforcement loading and consequently, higher strength. Woven glass fiber as a weight percent of laminate may be range to 65%. Woven roving are plainly woven from roving, with higher dimensional properties and regular distribution of glass fiber with excellent bonding strength among laminates possesses higher fiber content, tensile strength, impact resistance. The combination of different materials has been used for many thousands of years to achieve better performance requirements. There are nowadays many examples in the aeronautical and automobile industries, and yet the application of composite materials is still growing, including now areas such as aeronautical industries, sporting goods, civil and aerospace construction.

Recent developments in textile manufacturing processes show some promise in overcoming the above limitations. Textile process such as weaving, braiding and knitting can turn large volumes of yarn into dry preforms at a faster rate, thus reducing costs and cycle times. The dry preforms are impregnated with an appropriate matrix material and cured in a mold by

using processes such as resin-transfer molding (RTM). Two-dimensional woven and braided mats offer increased through-thickness properties as a consequence of yarn interlacing. The mats may be stitched with Kevlar or glass threads to provide additional reinforcement in the thickness direction. Three-dimensional woven and braided composites provide multidirectional reinforcement, thus directly enhancing the strength and stiffness in the thickness direction. Unlike laminated structures, three dimensional composites do not possess weak planes of de-lamination, thus giving increased impact resistance and fracture toughness. Textile manufacturing processes in conjunction with resin-transfer molding are also suitable for the production of intricate structural forms with reduced cycle times. This allows complex-shaped structures to be fabricated as integral units, thus eliminating the use of joints and fasteners.

With the advancements in the aforementioned technologies there is a need to develop scientific methods of predicting the performance of the composites made by the above processes. There are numerous variables involved in textile processes besides the choice of the fiber and matrix materials. This, for example, includes (1) the number of filaments in the yarn specified by the yarn linear density and (2) the yarn architecture (description of the yarn geometry) determined by the type of weaving or braiding process. Thus, there is a need for analytical/numerical models to study the effect of these variables on the textile composite behavior. Ideally, a structural engineer would like to model textile composites as a homogeneous anisotropic material-preferably orthotropic-so that the structural computations can be simplified, and also the existing computer codes can be used in the design. This would require the prediction of the effective (macroscopic) properties of the composites from the constituent material (microscopic) characteristics such as yarn and matrix properties, yarn/matrix interface characteristics and the yarn architecture. This is possible if we assume that there is a representative volume element (RVE) or a unit cell that repeats itself throughout the volume of the composite, which is true in the case of textile composites. The unit cell can be considered as the smallest possible building block for the textile composite, such that the composite can be created by assembling the unit cell in all three dimensions. The prediction of the effective macroscopic properties from the constituent material characteristics is one of the aspects of the science known as ‘micromechanics’. The effective properties include

thermo-mechanical properties like stiffness, strength and coefficients of thermal expansion as well as thermal conductivities, electromagnetic and other transport properties.

The increasing use of composite materials has revolutionized the aerospace industry over the past two decades. The ability to vary the properties and performance of composite materials has been in large measure responsible for the great impact that these materials have had. Traditionally, advanced composite structures have been fabricated from tape prepregs which were systematically stacked to form a laminate. This type of construction tends to give optimal in-plane stiffness and strength. Since the primary loads usually are in-plane, the use of such composites appeared logical. However, there are many situations where neither primary nor secondary loads are in-plane. In such situations tape prepreg laminates may not be the most appropriate. The future for composites is undergoing a transition. The aerospace performance criteria consisting of high specific stiffness and high specific strength are being supplemented with high toughness and efficient manufacturability. With this, textile structural composites in general and woven fabric (WF) composites in particular are finding increasing use in primary as well as secondary structural applications along with unidirectional (UD) tape composites. Making use of the unique combination of light weight, flexibility, strength and toughness, textile structures like wovens, knits, braids and nonwovens have now been recognized as attractive reinforcements for structural applications. Woven fabric is formed by interlacing two mutually perpendicular sets of yarns. The lengthwise threads are called warp and the crosswise threads fill or weft. The interlacing pattern of the warp and fill is known as the weave. Two-dimensional (2-D) fundamental weaves are plain, twill and satin. The micromechanical behavior of woven fabric laminates depends on the fabric properties, which in turn depend on the fabric structure. The parameters involved in determining the fabric structure are weaving, fabric count, fineness of yarn, fiber characteristics, yarn structure, degree of undulation, etc. The architecture of a WF lamina is complex and therefore the parameters controlling the mechanical and thermal properties of WF composites are too numerous. This makes it impractical to characterize the WF composites through tests alone, necessitating analytical models which can predict mechanical and thermal properties of the WF composites.

In the present work, the effective elastic properties of plain woven lamina are determined by the analytical method. Here the classical laminate theory (CLT) is used to determine the effective elastic properties of orthogonal plain weave fabric laminae. By using the CLT in-plane properties have been determined. CLT does not account for transverse shear deformation. The transverse shear strains, hence transverse shear deformations can be determined by first order shear deformation theory (FSDT).

Chapter 2

**REVIEW OF
LITERATURE**

REVIEW OF LITERATURE

Intensive studies have been done to investigate the mechanical properties of textile composites. Because woven fabric composites are the most often used form among textile fabric composites in structural applications, most of the previous works have mainly focused on woven fabric composites. Among them, Ishikawa and Chou [4] developed three analytical models for 2D woven composites based on classical lamination theory (CLT): the mosaic model, the fiber crimp (or undulation) model, and the fiber bridging model. As compared with experimental data, it is shown that the mosaic model provides a rough but convenient estimate of the elastic properties of fabric composites, the crimp model is suitable for plain weave fabrics and the bridging model is desirable for satin weave fabrics.

An analytical method proposed by Bhavani V. Sankar, Ramesh V. Marrey [5] called the selective averaging method (SAM) is proposed for prediction of the thermoelastic constants of textile composite materials. The unit cell of the composite is divided into slices (mesoscale), and the slices are subdivided into elements (microscale). The elastic constants of the homogenized medium are found by averaging the elastic constants of the elements selectively for both isostress and isostrain conditions. For thin textile composites where there are fewer unit cells in the thickness direction, SAM is used to compute directly the $[A]$, $[B]$ and $[D]$ matrices of the composite plate.

Rajiv A. Naik [6] proposed an analytical method in which the yarns are discretised into segments. Knowing the direction of the yarn in each segment, the segment stiffness is computed by using appropriate transformations. Then, assuming a state of isostrain, the textile composite stiffness is obtained by volume averaging the yarn-segment stiffness and matrix stiffness in the unit cell. This method seems to work when there is multidirectional reinforcement in the composite.

An Analytical and Experimental analysis proposed by N. K. Naik and V. K. Ganesh [7]. two fabric composite models for the on-axes elastic analysis of two-dimensional orthogonal plain weave fabric lamina. These are two dimensional models taking into account the actual strand cross-section geometry, possible gap between two adjacent strands and undulation and

continuity of strands along both warp and fill directions. The shape functions considered to define the geometry of the woven fabric lamina compare well with the photomicrographs of actual woven fabric lamina cross-sections. There is a good correlation between the predicted results and the experimental values. Certain modifications are suggested to the simple models available in the literature so that these models can also be used to predict the elastic properties of woven fabric laminae under specific conditions. Some design studies have been carried out for graphite/epoxy woven fabric laminae. Effects of woven fabric geometrical parameters on the elastic properties of the laminae have been investigated.

A two-dimensional woven fabric composite strength model is presented for the prediction of failure strength of two-dimensional orthogonal plain weave fabric laminates under on-axis uniaxial static tensile loading by N. K. Naik & V. K. Ganesh[12]. Different stages of failure such as warp strand transverse failure, fill strand shear/transverse failure, pure matrix block failure and the failure of matrix and fiber in the fill strand in longitudinal tension are considered. Material and geometrical nonlinearities have been considered for predicting the stress-strain behavior. The studies were carried out for three idealized laminate configurations. The possible shift of layers with respect to each other along x-, Y- and -directions were considered for the laminates.

N. K. Naik & V. K. Ganesh [3] developed, Three plain weave fabric composite analysis models for the prediction of the on-axes thermal expansion coefficients. These are two-dimensional models in the sense that the actual strand cross-sectional geometry, strand undulation and the presence of a gap between the adjacent strands are taken into account. In the first two models, termed refined models, the representative unit cell is discretized into slices and elements and analyzed. In the third method, a closed-form solution is presented. In this case, the representative unit cell is idealized as a cross ply laminate and analyzed. The relative merits and demerits of the models are also discussed. The predicted results are compared with the experimental values. A good correlation is observed.

A finite element model of polymer composites with three-dimensional (3D) reinforcement proposed by B.N. Cox W.C. Carter and N.A. Fleck[10] The model performs Monte Carlo simulations of failure under monotonic and fatigue loading. The formulation of the model is

guided by extensive prior experimental observations of 3D woven composites. Special emphasis is placed on realistic representation of the pattern of reinforcing tows, random irregularity in tow positioning, randomness of the strengths of constituent elements, and the mechanics of stress redistribution around sites of local failure. The constitutive properties of model elements (or their distributions) are based on micromechanical models of observed failure events. Material properties that are appropriately analyzed by the model are contrasted with those amenable to much simpler models.

A unified and rational treatment of the theory of fiber reinforced composite materials is presented by Hashin Z [26]. Fundamental geometric and elasticity considerations are thoroughly covered, and detailed derivations of the effective elastic moduli for these materials are presented. Biaxially reinforced materials which take the form of laminates are then discussed. Based on the fundamentals presented in the first portion of this volume, the theory of fiber-reinforced composite materials is extended to include visco-elastic and thermo-elastic properties. Thermal and electrical conduction, electrostatics and magneto-statics behavior of these materials are discussed. Finally, a brief statement of the very difficult subject of physical strength is included.

The effects of thermal expansion coefficient differentials between the reinforcing fiber and matrix materials of composites are discussed by Kabelka J [9]. Passing from the simpler case of unidirectional composites to those of angle ply laminates and 'balanced laminae'. Isotropic layers and woven fabrics are also treated. Attention is given to the important factor of residual stresses in composites, which arise in the course of their fabrication and may shorten the service life of a component or structure; the assessment of these stresses at the micro residual and macro residual levels are dealt with in detail. Mathematical expressions are given for the computation of important thermal parameters

An analytical technique using a plain weave classical laminate theory was used to predict the elastic properties of ceramic matrix woven fabric composites by K. Ranji Vaidyanathan, Ajit D. Kelkar, Jagannathan Sanka [10]. The model was developed by considering a typical representative fabric element, within which a repeating unit, the unit cell, was identified. An analytical procedure was developed to determine the elastic properties of a single plain weave

composite using the material properties of the constituents. These properties were then used to predict the elastic properties of ceramic laminates fabricated by stacking plain weave plies with differing orientations.

A practical computational procedure based on a global / local finite element method was developed by Johan D Whitcomb [11]. This procedure utilizes two problem levels: global and local levels. At the global level, an initial global solution was obtained using a coarse global mesh. At the local level, a small portion of the textile composite was modeled with a refined local mesh. For global analysis, macroelement used since the use of effective engineering properties are not in general accurate for the larger microstructure scale found in textile composites.

Three analytical models for the investigation of the stiffness and strength of woven fabric composites has been presented by Ishikawa, T. & Chou, T. W [4]. The “mosaic model” is effective in predicting the elastic properties of fabric composites. The “fibre undulation model” takes into account fiber continuity and undulation and has been adopted for modeling the “knee behavior” of plain weave fabric composites. The “bridging model” is developed to simulate the load transfer among the interlaced regions in satin composites. The theoretical predictions coincide extremely well with experimental measurements. The elastic stiffness and knee stress in satin composites are higher than those in plain weave composites due to the presence of the bridging regions in the weaving pattern.

An analytical method for Elastic Behavior of Woven Hybrid Composites proposed by Ishikawa, T. & Chou, T. W [15]. Basic geometrical and material parameters are identified to characterize the structure of hybrid fabrics. Analysis of the elastic behavior is made based upon a mosaic model and the fabric composite can be regarded as an assemblage of asymmetrical cross ply laminates. Upper and lower bounds of elastic properties have been obtained and the results compare very favorably with experiments. The influence of fabric parameters on the elastic behavior has been demonstrated especially for the bending-stretching coupling effect. Essential considerations for fabric design also have been discussed.

The effective coefficients of piezoelectric fiber-reinforced composites (PFRC) though micromechanical analyzed by Mallik, N., and Ray, M. C [20]. The method of cells (MOC) and the strength of materials (SM) approach have been employed to predict the coefficients. A constant electric field is considered in the direction transverse to the fiber direction and is assumed to be the same both in the fiber and the matrix. MOC and SM predictions for the effective piezoelectric coefficient of the PFRC assessing the actuating capability in the fiber direction are in excellent agreement. It has been found for the piezoelectric fibers considered that, when the fiber volume fraction exceeds a critical fiber volume fraction, this effective piezoelectric coefficient becomes significantly larger than the corresponding coefficient of the piezoelectric material of the fiber. The methods also show the excellent matching of the predictions of the effective elastic constants and the dielectric constant of the PFRC in the useful range of fiber volume fraction.

Mallik, N., and Ray, M. C [21] have been done Static analysis of laminated smart composite plates integrated with a piezoelectric fiber-reinforced composite (PFRC) layer acting as distributed actuators has been carried out by a generalized-energy-based finite element model. A simple first-order shear deformation theory is used for deriving the model. Eight noded isoparametric serendipity elements are used for discretizing the domain. The performance of the PFRC layer has been investigated for both symmetric and antisymmetric cross-ply and antisymmetric angle-ply laminated composite shell substrates. The effect of piezoelectric fiber orientation on the control authority of the PFRC layer has also been studied.

A general purpose micromechanics analysis that discretely models the yarn architecture within the textile repeating unit cell was developed by Naik, R. A. [18]. To predict overall, three-dimensional, thermal and mechanical properties, damage initiation and progression, and strength. This analytical technique was implemented in a user-friendly, personal computer-based, menu-driven code called Textile Composite Analysis for Design (TEXCAD). TEXCAD was used to analyze plain weave and 2 x 2, 2-D triaxial braided composites. The calculated tension, compression, and shear strengths correlated well with available test data for both woven and braided composites. Parametric studies were performed on both woven and braided architectures to investigate the effects of parameters such as yarn size, yarn

spacing, yarn crimp, braid angle, and overall fiber volume fraction on the strength properties of the textile composite.

The influence of crack formation, residual thermal stresses, and weave curvature on the mechanical performance of G10-CR glass/epoxy laminates has been studied by Kriz, R. D. [19]. Improved material performance is suggested by studying the load-deformation response of a unit cell of plain weave. A generalized plane strain finite-element model was used to predict crack-tip singularities and redistribution of stresses within a thin slice of warp-fill fiber bundles. The model predicts that warp curvature and thermal stresses at low temperatures are beneficial in reducing crack-tip singularities of fill cracks. The opposite is true for stiffness, which is decreased both by curvature and fill cracks. Results of this model provide the designer a tradeoff between stiffness and strength.

Naik, R. A., Ifju, P. G. and Masters, J. E [17] has been analyzed, the effects of various braiding parameters for triaxially braided textile composites were systematically investigated both experimentally and analytically. Four different fiber architectures designed to provide a direct comparison of the effects of braid angle, yarn size and axial yarn content were tested. Moiré interferometry was employed to study the effect of these parameters on the surface strain fields in the material.

An analytical method proposed by Ishikawa, T. & Chou, T. W [16]. In-plane thermal expansion coefficients and thermal bending coefficients of fabric composites. Three physical models have been adopted. The "mosaic model" provides a simple means for estimating these thermal properties. The one-dimensional "fiber undulation model" and the two-dimensional "bridging model" are suitable in particular for analyzing the thermo-mechanical behavior of plain weave and satin weave composites, respectively. The experimental results on in-plane thermal expansion coefficients of a 5-harness satin composite agree well with the theory.

The upper and lower bounds of elastic stiffness and compliance constants of woven fabric composites are derived by Ishikawa, T. & Chou, T. W. [13], based upon a mosaic-like model as well as the assumptions of constant stress and constant strain. An approximate analysis taking into account fiber undulation and continuity also is conducted. Fiber undulation leads to a slight softening of the in-plane stiffness and does not affect the stretching/bending coupling constants. A transverse shear deformation is adopted and modified to examine the one-dimensional bending response of fabric composites.

Flexural stiffness properties of a textile composite beam are obtained from a finite-element model of the unit cell by Bhavani V. Sankar, Ramesh V. Marrey [27] . Three linearly independent deformations, namely, pure extension, pure bending and pure shear, are applied to the unit cell. The top and bottom surfaces of the beam are assumed to be traction free. Periodic boundary conditions on the lateral boundaries of the unit cell are enforced by multi-point constraint elements. From the forces acting on the unit cell, the flexural stiffness coefficients of the composite beam are obtained. The difficulties in determining the transverse shear stiffness are discussed, and a modified approach is presented. The methods are first verified by applying them to isotropic and bimaterial beams for which the results are known, and then illustrated for a simple plain-weave textile composite.

Chapter 3

**THEORY AND
COMPUTATIONAL
WORK**

THEORY AND COMPUTATIONAL WORK

3.1 INTRODUCTION OF PLAIN WEAVE LAMINA

A single layer WF composite is designated as WF lamina. The woven fabric can be in the form of an open weave or a close weave. In the case of the open weave, there may be gaps between two adjacent strands, whereas close weave fabrics are tightly woven without any gap between two adjacent strands. There can also be certain fabrics made of twisted strands which would invariably have a certain amount of gap even if they are tightly woven. It is obvious that the presence of a gap between the adjacent strands would affect the stiffness of the WF lamina and hence should be accounted for while evaluating the mechanical properties. The experimentally determined fiber volume fraction, V_f , of the WF lamina is the overall $V_f V_f^0$, but for the analysis of the WF lamina the strand V_f , V_f^s forms the input. It is therefore necessary to evaluate the strand V_f from the overall V_f determined experimentally. The available methodologies do not take into account the gap between the adjacent strands, the actual cross-sectional geometry of the strand, and strand undulation transverse to the loading direction.



Figure3.1 Plain weave fabric structure

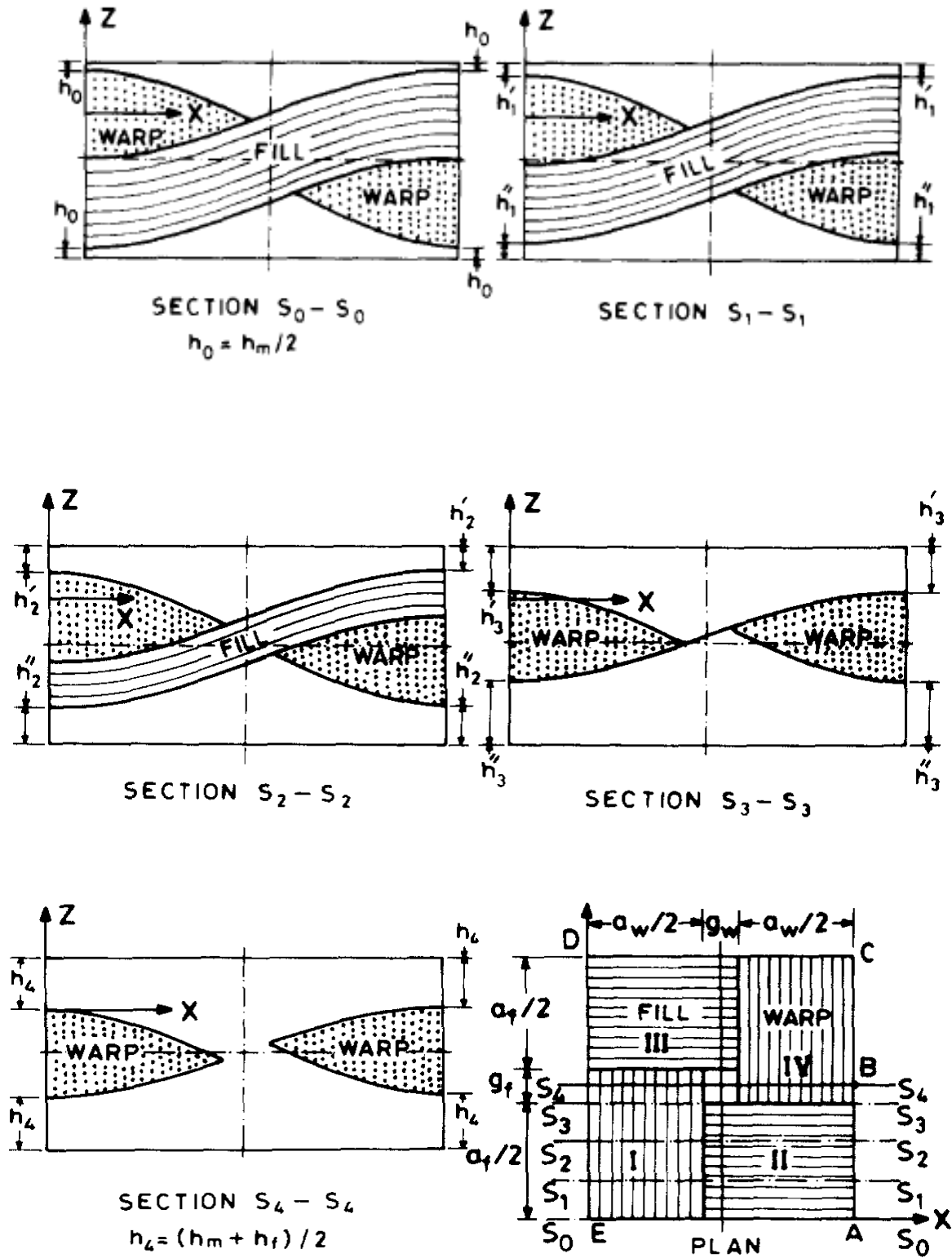


Figure 3.2 Plain weave fabric lamina structures – cross sections at different intervals.

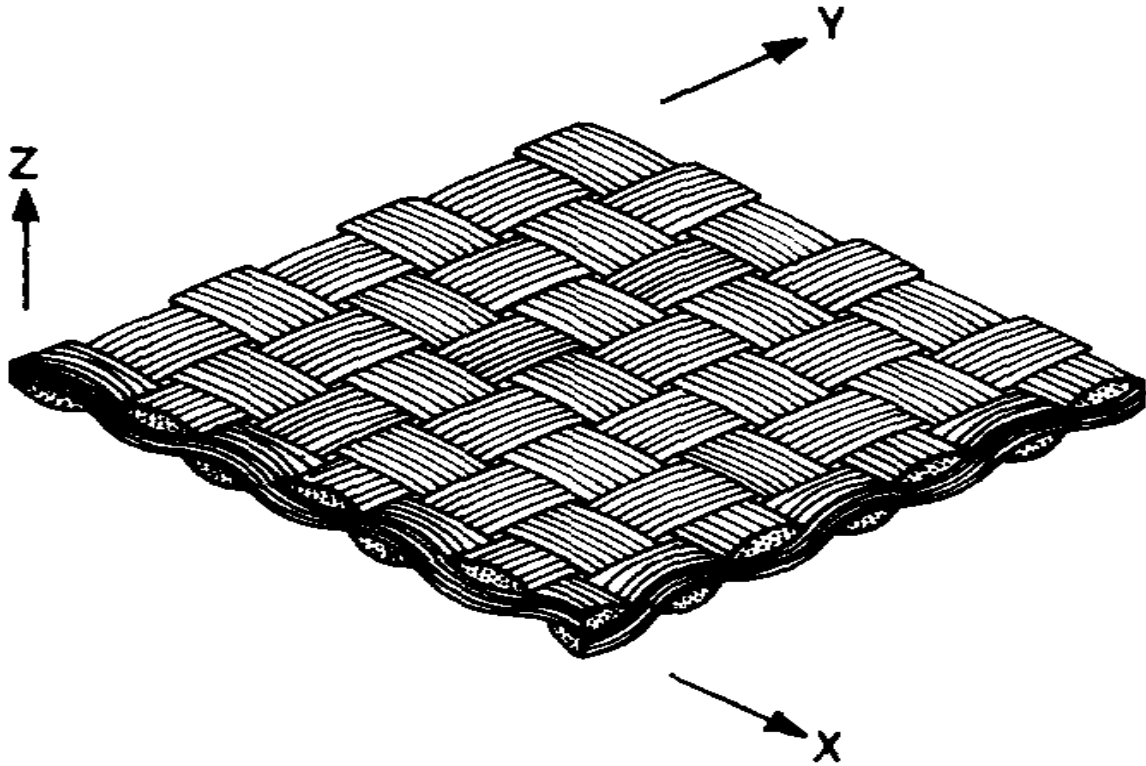


Figure 3.3 3D Plain weave fabric structure.

A typical plain weave fabric structure is shown in fig 3.3. Fig 3.2 present the cross-sections of plain weave fabric lamina at different sections from the midpoint of fill strand (S_0-S_0) to the midpoint of gap (S_4-S_4). It is seen that the thickness of the fill strand decreases gradually from the midpoint of the strand to zero in the gap region. This reduction due to the strand cross-sectional geometry would reduce the overall stiffness of the WF lamina. Therefore, the geometry of the strand cross-section should be considered while evaluating the stiffness and this requires a 2-D model. The available 1-D models predict higher stiffness as the maximum strand thickness is considered in these models.

3.2 MATERIAL USED IN ANALYSIS

There are two composite materials (E- Glass/ epoxy and T-300 carbon/ epoxy) used in analysis, the ingredient of these composite materials are as follow –

3.2.1 E-GLASS FIBER

Fiberglass, (also called **fiberglass** and **glass fiber**), is material made from extremely fine fibers of glass. It is used as a reinforcing agent for many polymer products; the resulting composite material, properly known as fiber-reinforced polymer (FRP) or glass-reinforced plastic (GRP), is called "fiberglass" in popular usage. Glassmakers throughout history have experimented with glass fibers, but mass manufacture of fiberglass was only made possible with the invention of finer machine tooling

Glass fiber is formed when thin strands of silica-based or other formulation glass is extruded into many fibers with small diameters suitable for textile processing. The technique of heating and drawing glass into fine fibers has been known for millennia; however, the use of these fibers for textile applications is more recent. Until this time all fiberglass had been manufactured as staple. When the two companies joined to produce and promote fiberglass, they introduced continuous filament glass fibers.

3.2.2 T-300 CARBON

Carbon fiber (alternatively called carbon fiber, graphite fiber, and graphite fiber or carbon graphite) is a material consisting of extremely thin fibers about 0.005–0.010 mm in diameter and composed mostly of carbon atoms. The carbon atoms are bonded together in microscopic crystals that are more or less aligned parallel to the long axis of the fiber. The crystal alignment makes the fiber very strong for its size. Several thousand carbon fibers are twisted together to form a yarn, which may be used by itself or woven into a fabric. Carbon fiber has many different weave patterns and can be combined with a plastic resin and wound or molded to form composite materials such as carbon fiber reinforced plastic (also referenced as carbon fiber) to provide a high strength-to-weight ratio material. The density of carbon fiber is also considerably lower than the density of steel, making it ideal for applications requiring low

weight. The properties of carbon fiber such as high tensile strength, low weight, and low thermal expansion make it very popular in aerospace, civil engineering, military, and motorsports, along with other competition sports.

Carbon fiber is most notably used to reinforce composite materials, particularly the class of materials known as Carbon fiber or graphite reinforced polymers. Non-polymer materials can also be used as the matrix for carbon fibers. Due to the formation of metal carbides (i.e., water-soluble AlC) and corrosion considerations, carbon has seen limited success in metal matrix composite applications. Reinforced carbon-carbon (RCC) consists of carbon fiber-reinforced graphite, and is used structurally in high-temperature applications. The fiber also finds use in filtration of high-temperature gasses, as an electrode with high surface area and impeccable corrosion resistance, and as an anti-static component. Molding a thin layer of carbon fibers significantly improves fire resistance of polymers or thermoset composites because dense, compact layer of carbon fibers efficiently reflects heat.

3.2.3 EPOXY

In chemistry, epoxy or poly-epoxide is a thermosetting epoxide polymer that cures (polymerizes and crosslinks) when mixed with a catalyzing agent or hardener. Most common epoxy resins are produced from a reaction between epichloro- hydrin and bisphenol-A.

Epoxy adhesives are a major part of the class of adhesives called "structural adhesives" or "engineering adhesives" (which also includes polyurethane, acrylic, cyano -acrylate, and other chemistries.) These high-performance adhesives are used in the construction of aircraft, automobiles, bicycles, boats, golf clubs, skis, snow boards, and other applications where high strength bonds are required. Epoxy adhesives can be developed to suit almost any application. They are exceptional adhesives for wood, metal, glass, stone, and some plastics. They can be made flexible or rigid, transparent or opaque /colored, fast setting or extremely slow setting. Epoxy adhesives are almost unmatched in heat and chemical resistance among common adhesives. In general, epoxy adhesives cured with heat will be more heat- and chemical-resistant than those cured at room temperature. The strength of epoxy adhesives is degraded at temperatures above 350°F.

3.3 FABRIC COMPOSITE MODELS

The plain weave fabric composite models presented here are 2-D in the sense that they consider the undulation and continuity of the strand in both the warp and fill directions. The models also account, for the presence of the gap between adjacent strands and different material and geometrical properties of the warp and fill strands.

3.3.1 Refined models

Two refined models are presented in this section. In the first model, the unit cell is discredited into slices along the loading direction. The individual slices are analyzed separately and the unit cell elastic properties are evaluated by assembling the slices under the isostrain condition. Such a model is called a slice array model, abbreviated SAM. In the second model, the unit cell is discredited into slices either along or across the loading direction. The slices are further subdivided into elements. The individual elements are analyzed separately. The elements are then assembled in parallel or series to obtain the slice elastic constants. Further, the slices are assembled either in series or parallel to obtain the elastic constants of the unit cell. This scheme of discrediting the unit cell into slices and further into elements is called an element array model, Abbreviated EAM.

(a) Slice array model (SAM)

In the analysis, the strand is taken to be transversely isotropic and its elastic properties are evaluated from the transversely isotropic fiber and matrix properties at strand V_f . It should be noted that owing to the presence of pure matrix pockets in the WF lamina, the strand V_f would be much higher than the composite overall V_f . The strand properties are evaluated using the composite cylinder assemblage (CCA) model (Reference 25 & 26). The representative unit cell of a WF lamina is taken as shown in Fig. 3.4(a). By virtue of the symmetry of the interlacing region in plain weave fabric, only one quarter of the interlacing region is analyzed. The analysis of the unit cell is then performed by dividing the unit cell into a number of slices as shown in Fig. 3.4(b). These slices are then idealized in the form of a four-layered laminate i.e. an asymmetric cross ply sandwiched between two pure matrix layers as shown in Fig. 3.4(c). The effective properties of the individual layer considering the presence of undulation are used to evaluate the elastic constants of the idealized laminate. This, in turn, is used to evaluate the elastic constants of the unit cell WF lamina.

(b) Element array model (EAM)

The limitations of SAM are that this method approximates the stiffness contribution of the warp strand and accounts for the gap between the adjacent warp strands approximately. It should also be noted that when the maximum off-axis angle Θ , is substantially high such that accurate enough to define the sine and cosine functions, SAM would fail to give accurate results.

In EAM these constraints are overcome by subdividing the slices into elements (1, 2, 3) of infinitesimal thickness (Fig. 3.5). Then, within these elements, the elastic constants of the warp and fill strands are transformed for the local off-axis angle (Fig. 3.5-3.6) and CLT is used to evaluate the stiffness of that element. The average in-plane compliance of the slices are evaluated under the constant stress condition in every element of that slice, i.e. the mean integral value of the element compliance over the length of the slice along the fill strand are evaluated. From the compliances of the slices the stiffnesses of the slices are calculated and then the elastic constants of the unit cell are evaluated considering a constant strain state in all the slices. This procedure where the elements in the slices are assembled in series (isostress condition) and then the slices are considered in parallel (isostrain condition) is one way of evaluating the overall stiffness (Fig. 3.5). Such a scheme is referred to as a series-parallel (SP) combination. The other way is to make the slices across the loading direction as shown in Fig. 3.6. The slices A', B' and C' are subdivided into elements. Then the elements in the slices A', B' and C' are assembled with isostrain condition to obtain the slice stiffness. The slice stiffnesses are inverted to obtain the slice compliances. The slices A', B' and C' are placed in series along the loading direction. The unit cell compliance is obtained by the integrated average of the slice compliances. The unit cell stiffnesses are obtained by inverting the unit cell compliances. Thus is the parallel series (PS) combination.

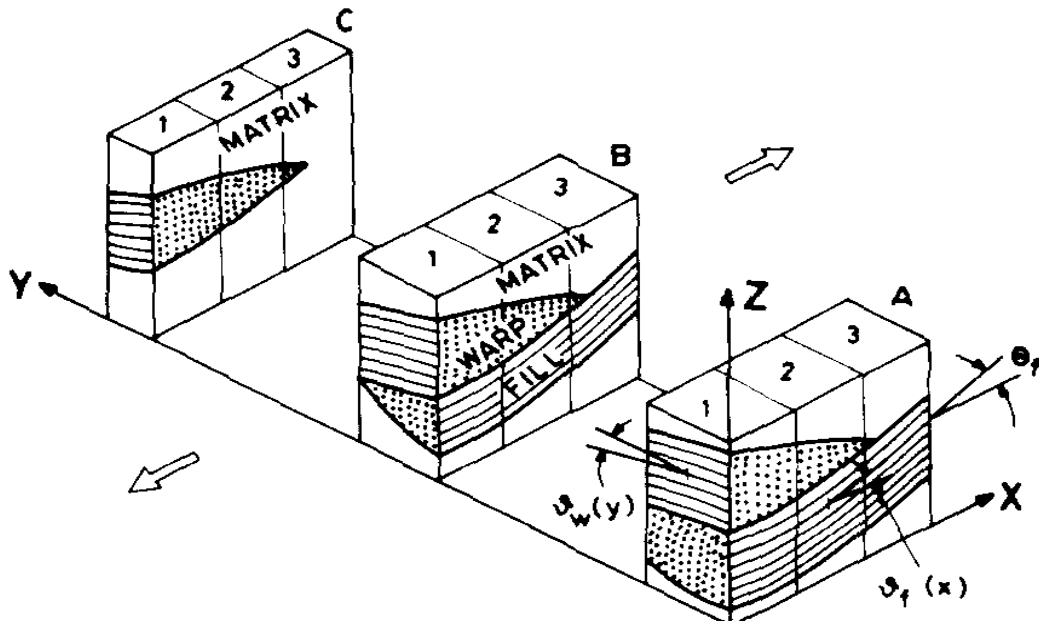


Figure 3.5 Series Parallel combination

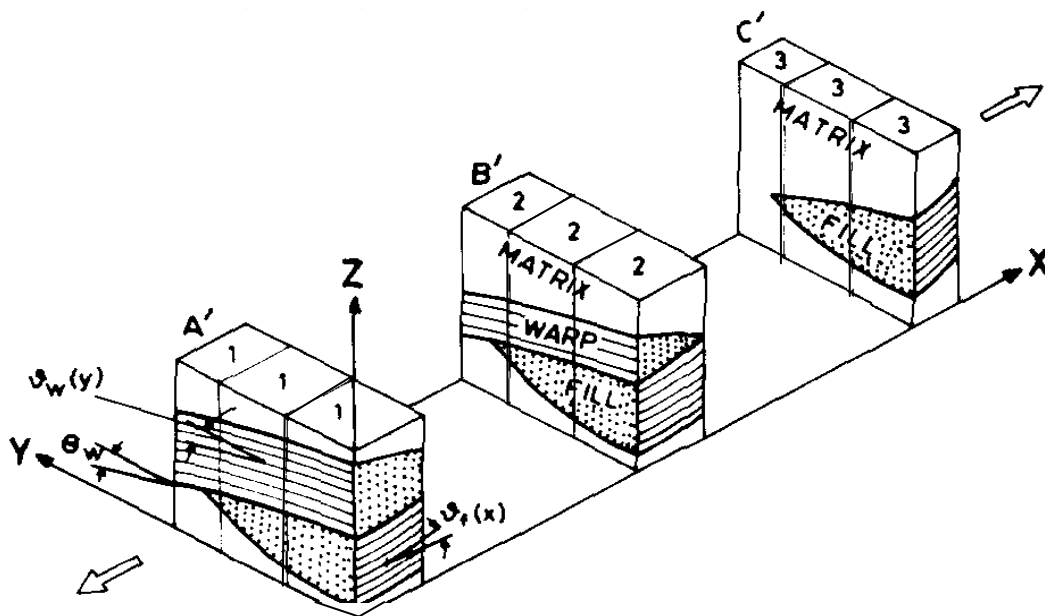


Figure 3.6 Parallel Series combination

3.4 COMPUTATION OF EFFECTIVE ELASTIC PROPERTIES

A single layer woven fabric (WF) composite is designated as a WF lamina. The representative unit cell of a WF lamina is shown in Figure 1a. Due to the symmetry of the interlacing region in plain weave fabrics, only one-quarter of the interlacing region is analyzed. To analyze the WF lamina, the unit cell is modeled as an asymmetric cross ply laminate composed of three layers, the layers being one pure matrix layer and two UD laminae as shown in Figure 1b. In order to determine the elastic properties of the WF lamina, it would be sufficient if the effective elastic properties of the idealized cross-ply laminate were determined. Each equivalent UD lamina of the cross-ply laminate represents one strand in the interlacing region. Therefore, to determine the elastic properties of the cross-ply laminate, the elastic properties of the strands forming the cross-ply are necessary.

The strand cross-section has a quasi-elliptical shape and the fibers in the strand are undulated in the longitudinal direction. Therefore, to compute the effective elastic properties of the strand, these two parameters i.e. the actual cross-section and the undulation – that predominantly affect the strand elastic properties should be considered. For the purpose of computation, the strand cross-section and the strand undulation are defined by suitable shape functions. The sinusoidal functions used to define the shape of the strand. From the undulation angle of the strands, the local reduced compliance constants of the strands are determined. These local reduced compliance constants are averaged over the length of the strand to determine the effective compliance of the strands. During averaging, the sinusoidal path of the strands in the unit cell is considered. Next, as an approximation, the variation of the strand undulation angle was considered to be linear. This means that the strand takes a circular path. The elastic properties of a WF lamina are determined with the assumption that the classical laminate theory is applicable in the unit cell and bending deformation of the unit cell are constrained by the adjacent unit cell in the case of plain weave fabric lamina. In view of the second assumption, the membrane stiffness constants are derived with zero curvature condition.

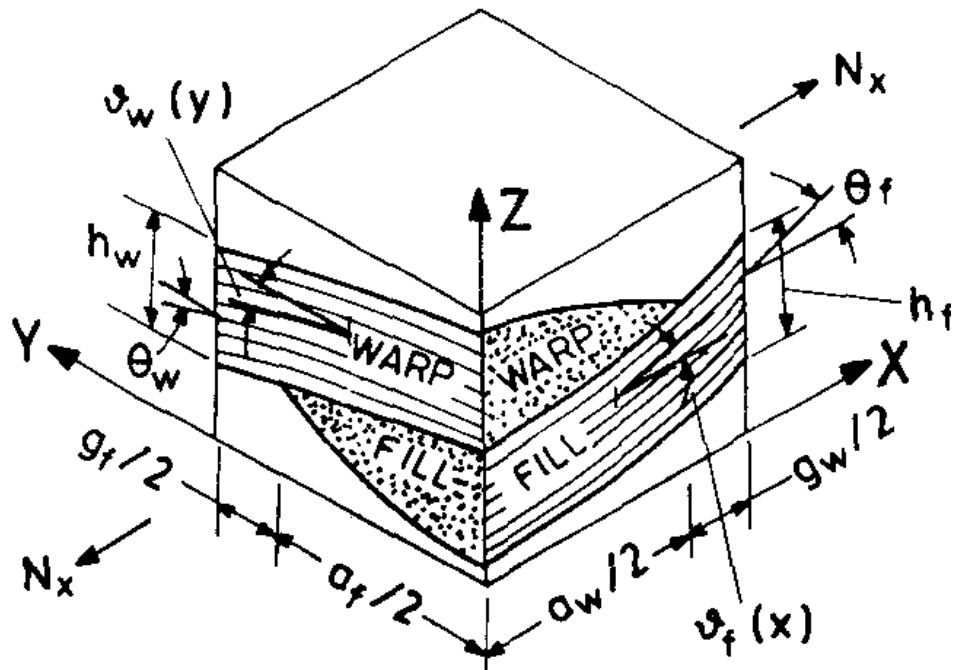


Figure 3.7 Actual unit cell of plain weave fabric lamina

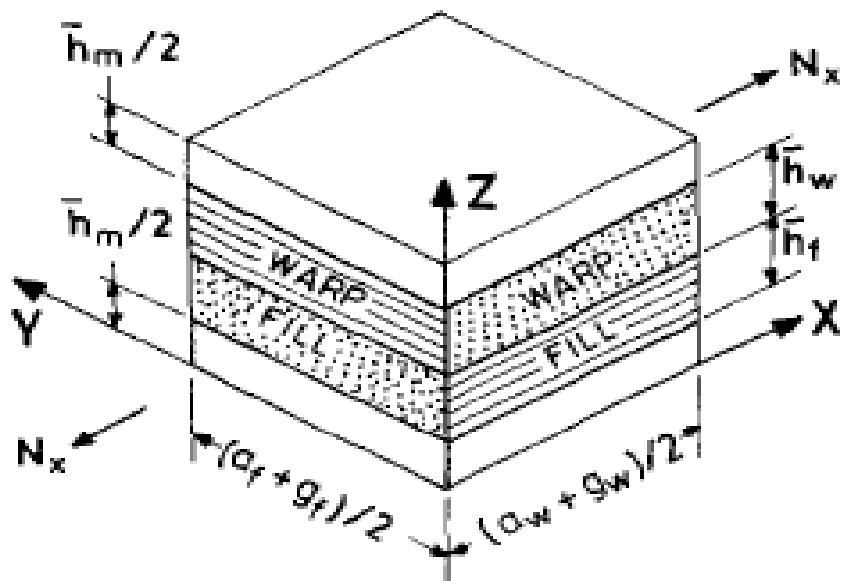


Figure 3.8 Ideal unit cell of plain weave fabric lamina

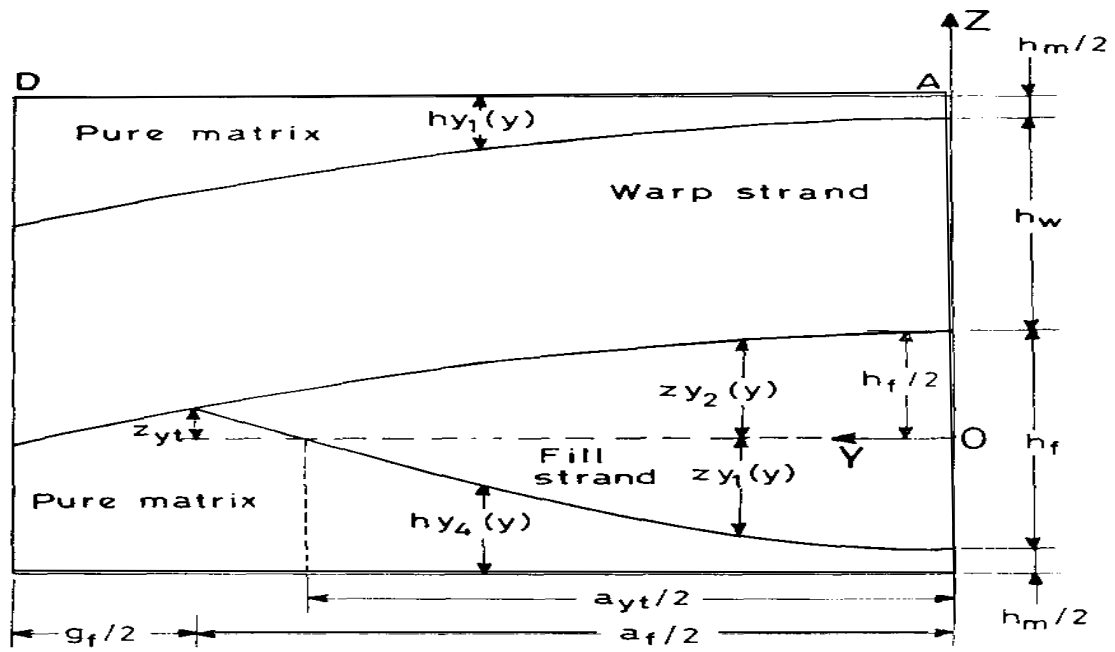


Figure 3.9 Plain weave fabric lamina cross –sections along warp direction

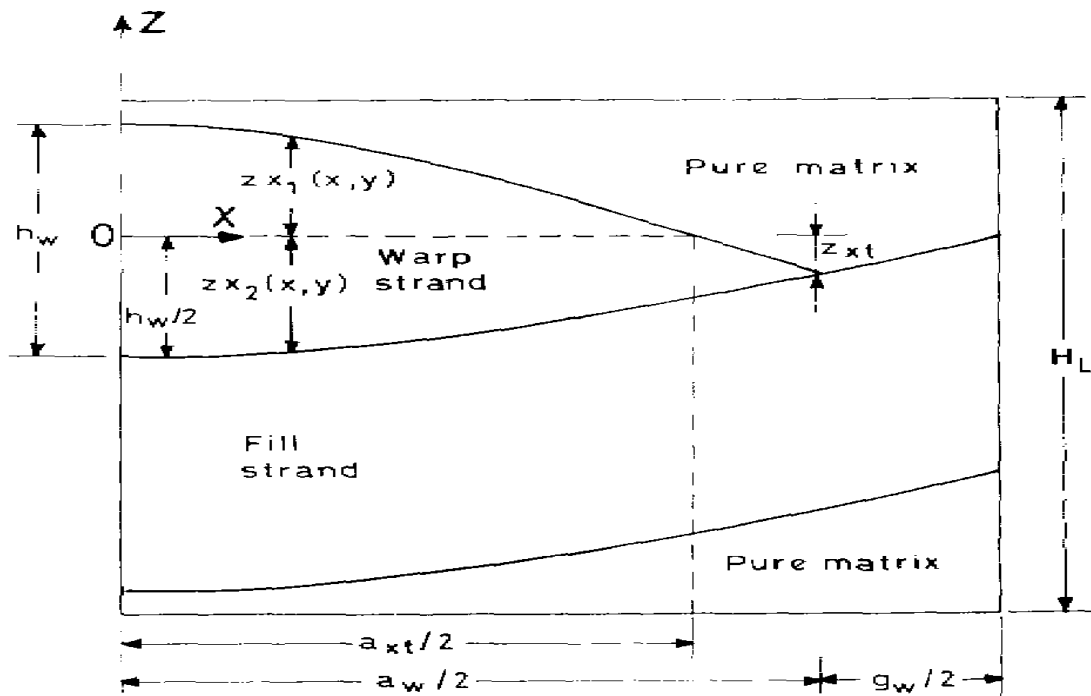


Figure 3.10 Plain weave fabric lamina cross-sections along fill direction

In the Y-Z plane, i.e. along the warp direction (Figure 3.9), the shape functions can be written as follows,

$$zy_1(y) = -\frac{h_f}{2} \cos\left(\frac{\pi y}{a_{yt}}\right) \quad (1)$$

$$zy_2(y) = \frac{h_f}{2} \cos\left(\frac{\pi y}{a_{yt}+g_f}\right) \quad (2)$$

where

$$a_{yt} = \frac{\pi a_f}{2[\pi - \cos^{-1}(2z_{yt}/h_f)]}$$

$$z_{yt} = \frac{h_f}{2} \cos\left[\frac{\pi a_f}{2(a_f+g_f)}\right]$$

and

$$hy_1(y) = \frac{h_f + h_m}{2} - zy_2(y) \quad (3)$$

$$hy_4(y) = \frac{h_f + h_m}{2} - zy_1(y) \quad (4)$$

In the X-Z plane, i.e., along the fill direction (Figure 3.10), the shape functions are given by,

$$zx_1(x, y) = \frac{h_w}{2} \cos\left(\frac{\pi x}{a_{xt}}\right) - hy_1(y) + \frac{h_m}{2} \quad (5)$$

$$zx_2(x, y) = -\frac{h_f}{2} \cos\left(\frac{\pi x}{a_w+g_w}\right) - hy_1(y) + \frac{h_m}{2} \quad (6)$$

where

$$a_{xt} = \frac{\pi a_w}{2 \cos^{-1}(2z_{xt}/h_w)}$$

$$z_{xt} = -\frac{h_w}{2} \cos \left[\frac{\pi a_w}{2(a_w + g_w)} \right]$$

In the above equations, the presence of a gap between the strands has been considered using gap parameters a_{xt} , a_{yt} , z_{xt} and z_{xt} . The mean thickness of the strands over the length of unit cell including gap region and the undulation angle of the strands can be calculated using the equation (1) - (6). The mean thickness of fill strand is given by

$$\overline{h_f} = \frac{2}{a_f + g_f} \int_0^{a_f/2} [zy_2(y) - zy_1(y)] dy \quad (7)$$

By substituting the value of $zy_2(y)$ and $zy_1(y)$ from equations (1) and (2) into equation (7) We get,

$$\overline{h_f} = \frac{h_f}{\pi(a_f + g_f)} \left[(a_f + g_f) \sin \left(\frac{\pi a_f}{2(a_f + g_f)} \right) + a_{yt} \sin \left(\frac{\pi a_f}{2a_{yt}} \right) \right] \quad (8)$$

Similarly the mean thickness of the warp strand can be determined as,

$$\overline{h_w} = \frac{2}{(a_w + g_w)} \int_0^{a_w/2} [zx_1(x, y) - zx_2(x, y)] dx \quad (9)$$

By substituting the values of $zx_1(x, y)$ and $zx_2(x, y)$ from equation (5) and (6) into equation (9) we get,

$$\overline{h_w} = \frac{h_w}{\pi(a_w + g_w)} \left[(a_w + g_w) \sin \left(\frac{\pi a_w}{2(a_w + g_w)} \right) + a_{xt} \sin \left(\frac{\pi a_w}{2a_{xt}} \right) \right] \quad (10)$$

The undulation angle of fill strand can be determined as,

$$\tan \theta_f(x) = \frac{d}{dx} [zx_2(x, y)] \quad (11)$$

thus

$$\theta_f(x) = \tan^{-1} \left[\frac{\pi h_w}{2(a_w + g_w)} \sin \left(\frac{\pi x}{a_w + g_w} \right) \right] \quad (12)$$

The undulation angle of warp strand can be determined as,

$$\tan \theta_w(y) = \frac{d}{dy} [zy_2(y)] \quad (13)$$

thus

$$\theta_w(y) = \tan^{-1} \left[\frac{-\pi h_f}{2(a_f + g_f)} \sin \left(\frac{\pi y}{a_f + g_f} \right) \right] \quad (14)$$

The undulation angle of the strands, the local reduced compliance constants of the strands are determined as follow,

$$\overline{S_{11}^p} = m^4 S_{11} + (2S_{12} + S_{66})m^2 n^2 + n^4 S_{22}$$

$$\overline{S_{12}^p} = (S_{11} + S_{22} - S_{66})m^2 n^2 + (m^4 + n^4)S_{12}$$

$$\overline{S_{22}^p} = n^4 S_{11} + (2S_{12} + S_{66})m^2 n^2 + m^4 S_{22}$$

$$\overline{S_{66}^p} = 2(2S_{11} - 4S_{12} + 2S_{22} - S_{66})m^2 n^2 + (m^4 + n^4)S_{66}$$

$$\overline{S_{44}^p} = S_{11}m^2 + S_{55}n^2$$

$$\overline{S_{45}^p} = (S_{55} - S_{44})mn$$

$$\overline{S_{55}^p} = S_{55}m^2 + S_{44}n^2$$

$$\overline{S_{16}^p} = (2S_{11} - 2S_{12} - S_{66})m^2 n - (2S_{22} - 2S_{12} - S_{66})n^3 m$$

$$\overline{S_{26}^p} = (2S_{11} - 2S_{12} - S_{66})n^3 m - (2S_{22} - 2S_{12} - S_{66})m^3 n$$

$$m = \cos \theta_p(q) \quad n = \sin \theta_p(q)$$

where $p = f, w$ and $q = x, y$

Once we determined the local reduced compliance constants, these are averaged over the length of strand to get the effective compliance of the fill and warp strands. From the effective compliance constant, the elastic coefficients of the fill and warp strands are determined.

$$\left[\overline{S_{ij}^p}\right]_{avg} = \frac{2}{a_k + g_k} \int_0^{\frac{(a_k + g_k)}{2}} \left[\overline{S_{ij}} \theta_p(q)\right] dq \quad (15)$$

where $k=w, f$ $p=f, w$ $q= x, y$ and $i, j=1, 2, 6$ and $i, j=4, 5$

The elastic coefficient of matrix is,

$$\left[C_{ij}^p\right] = \left[\overline{S_{ij}^p}\right]^{-1}_{avg} \quad (16)$$

The reduced elastic coefficient matrix of the fill and warp strands are,

$$\left[C_{ij}^p\right] = [T]^T \left[C_{ij}^p\right] [T] \quad (17)$$

The elastic properties of a WF lamina are determined with the assumption that the first order shear deformation theory (FSDT) is applicable in the unit cell. As the unit cell is subjected to only in plane loading; only the extensional constants would appear in the unit cell stiffness matrix. Therefore, the equation for the unit cell stiffness constants are reduce to,

$$\left[A_{ij}\right] = \frac{1}{H_L} \left[\overline{C_{ij}^f} \overline{h_f} + \overline{C_{ij}^w} \overline{h_w} + \overline{C_{ij}^m} \overline{h_m}\right] \quad (18)$$

The effective compliance constants can be obtained by inverting the above stiffness matrix, i.e.

$$\left[S_{ij}\right]_{eff} = \left[A_{ij}\right]^{-1}$$

$$(E_x)_{eff} = \frac{1}{S_{11}}$$

$$(v_{xy})_{eff} = -S_{12}(E_y)_{eff}$$

$$(G_{xy})_{eff} = \frac{1}{S_{56}}$$

As an approximation, the variation of the strand undulation angle can be considered to be linear. This means that the strand takes a circular path. For this case, the expressions for the effective compliance constants of the fill and warp strands in the unit cell can be written as

$$[\overline{S_{ij}}]_k = \frac{1}{\theta_k} \int_0^{\theta_k} S_{ij}(\theta_k) d\theta \quad (19)$$

The models used by T. Ishikawa, T. W. Chou [3] either do not consider or consider only approximately the determination of strand fiber volume fraction (V_f^S) from the experimentally determined overall fiber volume fraction (V_f^0) of the WF composite. It should be noted that the accurate computation of strand fiber volume fraction is an essential ingredient for the accuracy of any fabric composite analysis model. In the present analysis of strand is idealized as an equivalent UD lamina. Therefore, in equation (15) E_L , E_T , ν_{LT} , G_{LT} , G_{TT} are the elastic properties of straight strand at strand fiber volume fraction (V_f^S). The strand volume fraction (V_f^S) for given overall volume fraction (V_f^0) can be determined as,

$$V_f^S = \frac{V_f^0 V^0}{V^0 - V^{pm}} \quad (20)$$

where

$$\begin{aligned} V^0 &= (a_f + g_f)(a_w + g_w)H_L \\ V^{pm} &= (a_f + g_f)(a_w + g_w) \overline{h_m} \\ \overline{h_m} &= H_L - (\overline{h_f} + \overline{h_w}) \end{aligned}$$

Chapter 4

RESULTS AND DISCUSSION

RESULTS AND DISCUSSION

4.2 ANALYTICAL RESULTS

Two fabric composite materials have been presented for elastic analysis of 2D orthogonal plain weave fabric laminae. There are elastic properties of material in fill direction and geometrical parameters of strand in both warp and fill direction.

Material		E_{fT} (GPa)	E_{fT} (GPa)	G_{fLT} (GPa)	G_{fTT} (GPa)	ν_{fLT}
Fibers	T-300 Carbon	230.0	40.0	24.0	24.0	0.26
	E-Glass	72.0	72.0	27.7	27.7	0.30
Matrix	Epoxy resin	3.5	3.5	1.3	1.3	0.35

Table 4.1 Elastic properties of fibers and matrix

Material	Fabric Thickness h_t (mm)	Fill strand			Warp strand			V_f^o	
		a_f (mm)	h_f (mm)	g_f (mm)	a_w (mm)	h_w (mm)	g_w (mm)		
	E-Glass / epoxy								
GLE1	0.085	0.40	0.043	0.25	0.40	0.043	0.25	0.28	
GLE2	0.095	0.45	0.048	0.30	0.45	0.048	0.30	0.23	
GLE3	0.180	0.62	0.090	0.10	0.68	0.090	0.04	0.43	
GLE4	0.220	0.86	0.110	0.00	0.84	0.110	0.02	0.46	
	T-300 Carbon / epoxy								
CE1	0.200	1.80	0.100	0.65	1.45	0.100	1.00	0.27	
CE2	0.200	1.45	0.100	1.00	1.80	0.100	0.65	0.27	
CE3	0.160	0.96	0.080	0.18	1.10	0.080	0.04	0.44	
CE4	0.160	1.10	0.080	0.04	0.96	0.080	0.18	0.44	

Table 4.2 Plain weave fabric lamina and weave geometrical parameter, $h_w = h_f = h_t/2$, $u/a = 1.0$

Material	V_f^s	Predicted, E_x		EAM-PS	Experimental, E_x (Ref. 3)	
		Sinusoidal	Circular		E_x	SD
GLE1	0.7470	23.273	23.542	21.7	22.4	1.0
GLE2	0.7311	22.982	23.133	21.3	22.8	1.2
GLE3	0.7501	20.314	20.596	21.2	21.5	1.0
GLE4	0.740	16.354	16.986	14.5	17.5	1.0
CE1	0.7672	55.638	55.961	56.7	60.3	2.1
CE2	0.7672	54.124	54.631	49.8	49.3	1.9
CE3	0.6566	36.773	38.442	42.0
CE4	0.6566	30.772	33.452	32.2	36.5	1.8

Table 4.3 Predicted and Experimental Young's moduli of plain weave fabric composites (GPa)

Material	V_f^s	Predicted, G_{xy}		EAM-PS	Experimental G_{xy} (range) (Ref. 3)	
		Sinusoidal	Circular		10°	$\pm 45^\circ$
GLE1	0.7470	4.450	4.471	4.460	8.33(7.7-8.5)	5.50(5.4-5.6)
GLE2	0.7311	4.511	4.541	4.329	8.33(7.7-8.5)	5.50(5.4-5.6)
GLE3	0.7501	4.523	4.566	4.320	6.89(6.6-7.1)	2.94(2.9-3.0)
GLE4	0.740	4.668	4.707	4.530
CE1	0.7672	2.833	2.8451	2.90	10(9.4-10.9)	2.94(2.9-3)
CE2	0.7672	2.835	2.8355	2.90
CE3	0.6566	2.852	2.8631	2.10
CE4	0.6566	2.891	2.955	2.15	10(9.4-10.9)	2.94(2.9-3)

Table 4.4 Predicted and Experimental Shear moduli of Plain weave fabric composites (GPa)

Material	V_f^s	Predicted, ν_{xy}		EAM-PS	Experimental, ν_{xy} (Ref. 3)
		Sinusoidal	Circular		
GLE1	0.7470	0.1211	0.1211	0.203	0.10
GLE2	0.7311	0.1222	0.1211	0.205
GLE3	0.7501	0.1262	0.1253	0.199
GLE4	0.7401	0.1251	0.1298	0.189
CE1	0.7672	0.0233	0.0233	0.110
CE2	0.7672	0.0339	0.0339	0.131
CE3	0.6566	0.0421	0.0398	0.156
CE4	0.6566	0.0923	0.0389	0.66

Table 4.5 Predicted and Experimental Poisson's ratios of plain weave fabric composites

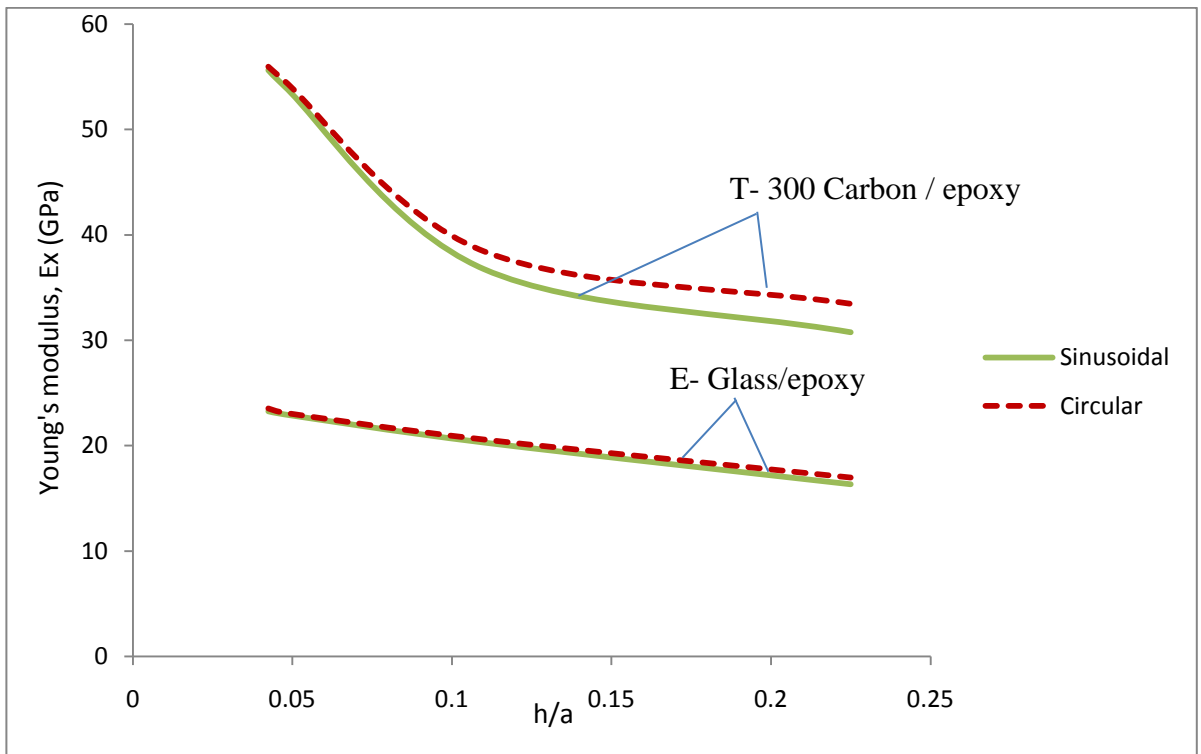


Figure 4.1 Variation of E_x as a function of h/a

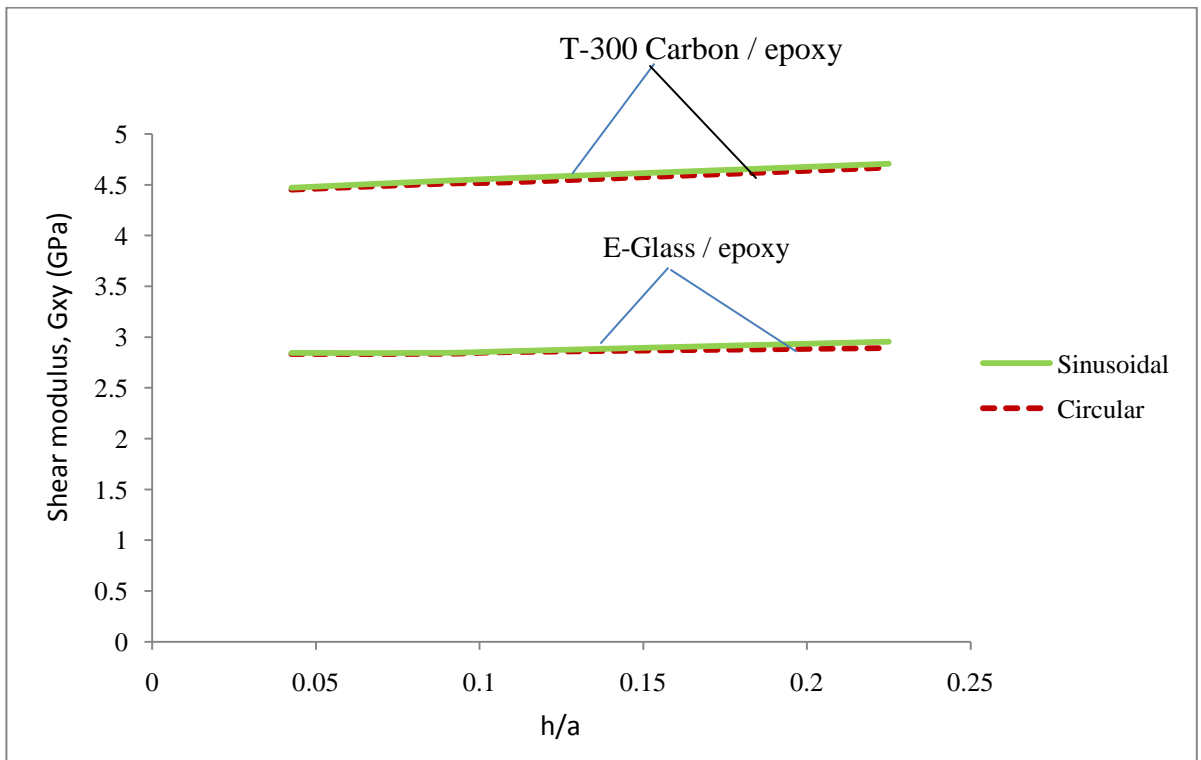


Figure 4.2 Variation of G_{xy} as a function of h/a

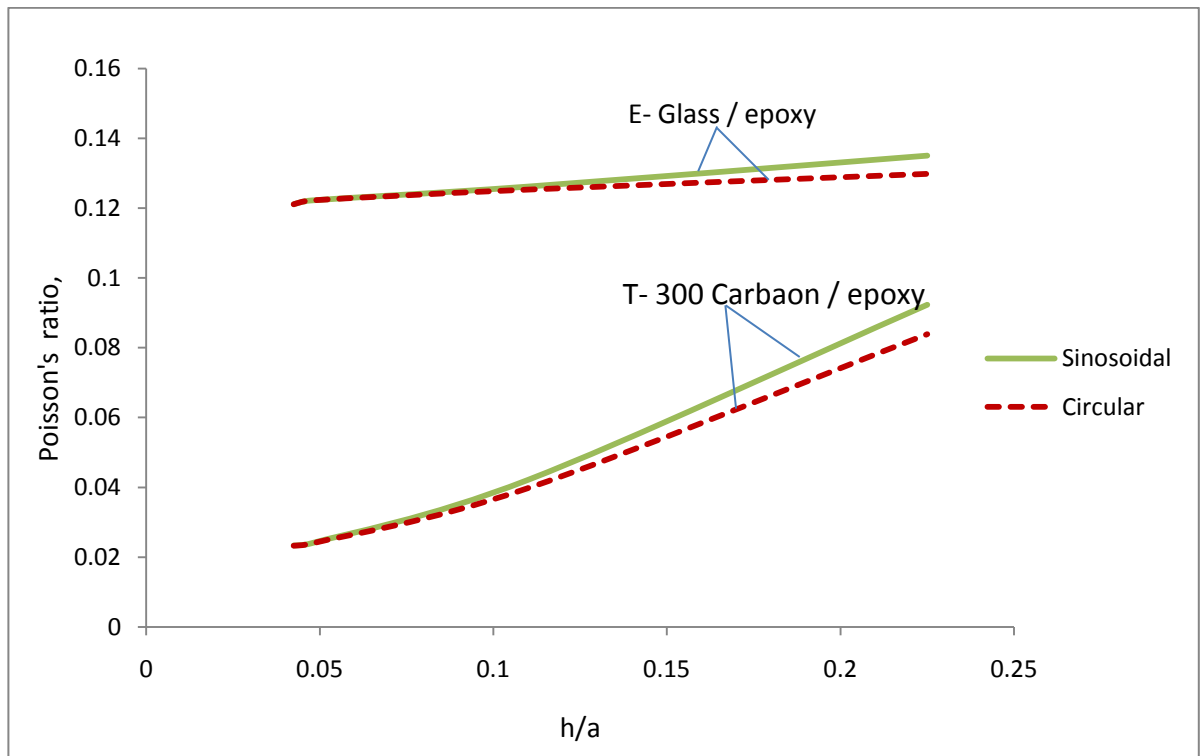


Figure 4.3 Variation of ν_{xy} as a function of h/a

4.2 DISCUSSION

The analytical method explained in the previous section has been used for the prediction of the elastic properties of plain weave fabric laminae. Eight material systems with different strand and weave geometries were considered for the analysis. The elastic properties of the fibers and matrix are given in Table 4.1. The corresponding strand and weave geometrical parameters are given in Table 4.2. The geometrical parameters were measured using an optical microscope. The experimentally determined V_f^o is also presented in Table 4.2. Equivalent UD lamina/strand properties were calculated using the composite cylinder assemblage model proposed by Hashin [26] for all the material systems at the corresponding V_f^s . Using these UD lamina/strand properties and the analytical method presented earlier, the elastic properties of the WF laminae were computed. The predicted and the experimentally determined elastic properties are presented in Tables 4.3-4.5. The effective compliance constants can be evaluated using either equation (15) or equation (19). By using equation (15), the strand path is assumed to be sinusoidal for the evaluation of the effective compliance constants. This approach is designated as sinusoidal. By using equation (19), the strand path is assumed to be circular for the evaluation of the effective compliance constants. This approach is designated as circular. It should be noted that in both the cases, the strand geometry is sinusoidal as given by equations (1)-(6). The results obtained using the element array model with parallel-series combination (EAM-PS), as given by Naik and Ganesh [7], are also presented.

Here the predicted value of elastic properties i.e. Young modulus, Shear modulus, and Poisson's ratios of plain weave fabric laminae has been determined by Matlab tool. All results presented in table 4.3-4.5 are with respect to the fill direction. From Table 4.3 it is seen that the circular path gives higher values than the sinusoidal path for Young's moduli. In the sinusoidal function, the variation of undulation is gradual initially, and later is steep. In the circular function, the variation of undulation is uniform throughout. Hence the circular path gives higher values of Young's moduli. As the sinusoidal function models the strand undulation more accurately, this approach is more realistic. In the closed-form analytical method the warp strand is assumed to be distributed uniformly in the unit cell. Hence it is

expected to bridge the gap region. Therefore, the predictions using the closed-form analytical method give higher values of Young's moduli than those predicted using the EAM-PS model [7]. As an exact weighting is given to each parameter affecting the predictions in the EAM-PS model, this model can be considered as the reference method. But this model is computationally extensive compared with the closed-form analytical method. As seen from Table 3, for practical fabrics, the difference between the predictions of the EAM-PS and closed-form analytical methods is marginal. Comparing the predictions based on the sinusoidal path with the experimental results (Table 4.3), overall, a good correlation is observed. It should be noted that the predictions are for laminae whereas the experimental results are for laminates. A large difference is seen between the predicted and experimental values for the material system CE 1. This is because of the compaction of the layers in the actual laminate, leading to higher experimental results. Compaction was observed for this laminate because of the large inter-strand gap.

In general, the observations for G_{xy} (Table 4.4), and ν_{xy} (Table 4.5) are the same as those for E_x (Table 4.3). As seen from these tables, for the practical range of fabric geometries, the analytical predictions using the sinusoidal path and circular path gave practically identical results for G_{xy} , and ν_{xy} . Comparing the analytical predictions for G_{xy} with the experimental results ($\pm 45^\circ$ off-axis tension test), a good correlation is observed for all the material systems except for CE1. In the case of Young's modulus, the stiffness of a warp strand in the x-direction is significantly less than that of a fill strand and hence its contribution to unit cell/WF lamina properties is less.. Overall, there is a good correlation between the prediction results from MATLAB Program and experimental results from literature.

The effect of h/a ratio on the elastic properties is presented in Figures 4.1-4.3 for the balanced plain weave fabric laminae. The material systems considered are as given in Table 1. For the balanced plain weave fabric, $a_w = a_f$, $h_w = h_f$ and $g_w = g_f$. It can be seen from Figure 4.1 that E_x decreases as h/a increases. This is fact that the effect of undulation increases with increasing h/a , leading to the reduction in E_x . The rate of decrease is greater for higher values of E_L/E_T ratio of the corresponding UD lamina/strand. The variation of G_{xy} as a function of h/a is presented in Figure 4.2. For T-300 carbon/epoxy G_{xy} is practically constant. For E-

glass/epoxy, G_{xy} increases marginally as h/a increases. This is because G_{TT} is higher than G_{LT} for this material system. As expected, higher values of ν_{xy} are obtained with increasing values of h/a and the rate of increase is greater for larger values of E_L/E_T (Figure 4.3). As explained earlier, the analytical predictions with circular path give higher values of Young's moduli than with sinusoidal path (Table 4.3 and Figure 4.2). At lower values of h/a , the difference is practically zero. At higher values of h/a , the difference increases as the h/a ratio increases. The same observation is also true for shear modulus except for E-glass/epoxy. For E-glass/epoxy, sinusoidal path gives higher value than circular path because, for this material system, the UD lamina/strand G_{TT} is higher than G_{LT} . Analytical predictions with sinusoidal path give higher values of Poisson's ratios than with circular path. At lower h/a ratios the difference is very marginal, whereas at higher ratios the difference is significant. The results obtained by the closed-form analytical method with sinusoidal path match well with the results obtained by the EAM-PS model and the experimental results. Considering the simplicity of this analytical method, it can be used as a convenient engineering tool. For balanced plain weave fabrics, the properties along the warp and fill directions are the same. Hence, the discussions about the elastic properties along the fill direction and those along the warp direction are the same.

Chapter 5

**CONCLUSION AND
SCOPE FOR FUTURE**

CONCLUSION AND SCOPE FOR FUTURE WORK

5.1 CONCLUSION:-

A two-dimensional analytical method has been proposed to predict the elastic properties of 2D orthogonal plain weave fabric laminae. The elastic properties have been predicted for four material systems of E-Glass/epoxy and four material system of T-300 carbon/epoxy. There is a good correlation between the predicted results with existing literature, for both sinusoidal and circular path. A significant effect of h/a ratio on the elastic properties of the woven fabric laminae has been seen. Although the method assumes non-twisted strands, the effect of twist in the strands can also be considered in this method by multiplying the straight strand elastic properties by the fiber-to-strand property translation efficiency factor.

5.2 SCOPE FOR FUTURE:-

- Elastic properties can be predicted for plain weave fabric composite made of twist yarn.
- Elastic properties can be predicted for twill and satin weave composite.
- Elastic properties can be predicted for braided fabric composite.
- Vibration analysis can be done for Plain weave fabric composite models.

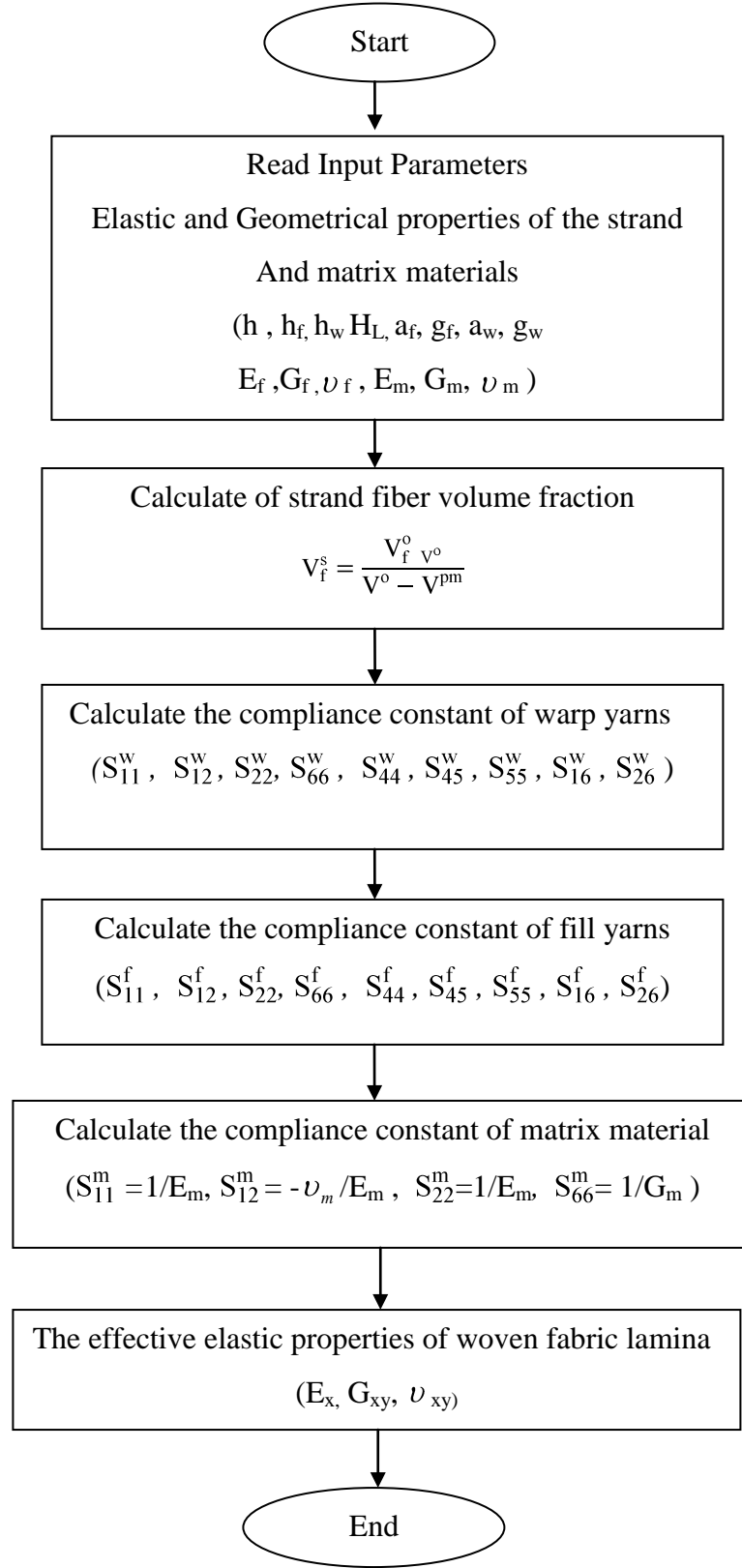
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APPENDIX



Flow chart for MATLAB Program